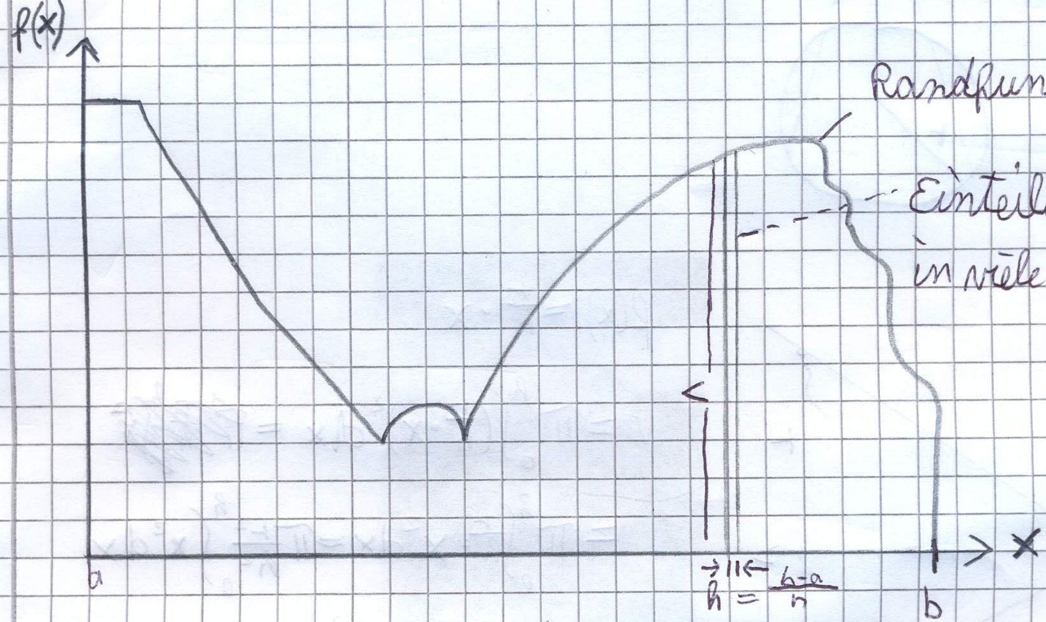
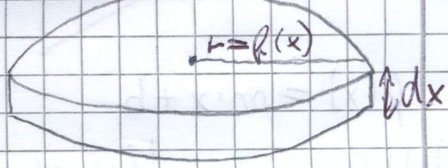
Fortsetzung des Protokolls Anthony Dorst

Volumen von Rotationskörpern

Pokal Querschnittsfläche an jeder Höhe: Kreis



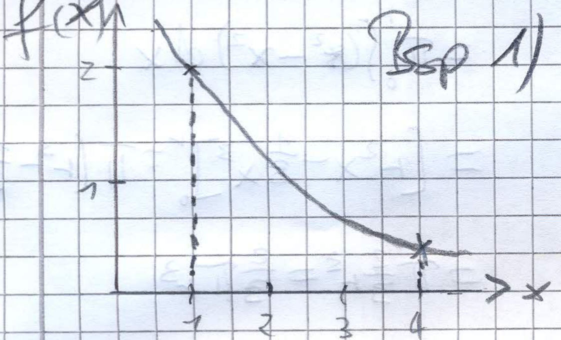
eine Scheibe: Zylinder mit:



$\Rightarrow \quad r = f(x) \qquad h = dx$

$$V = \pi \cdot r^2 \cdot h = \pi [f(x)]^2 \cdot dx$$

$$V_{ges} = \pi \int_a^b [f(x)]^2 \, dx$$

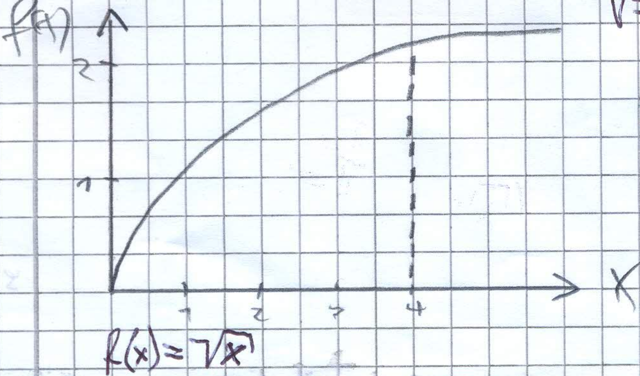


$f(x) = \frac{2}{x}$

Bsp 1) $V = \pi \int_1^4 \left(\frac{2}{x}\right)^2 dx = \pi \int_1^4 \frac{4}{x^2} dx = \pi \left[-\frac{4}{x}\right]_1^4$

$= \pi \left(\left(-\frac{4}{4}\right) - \left(-\frac{4}{1}\right)\right) = \pi(-1+4) = \underline{3\pi}$

2)



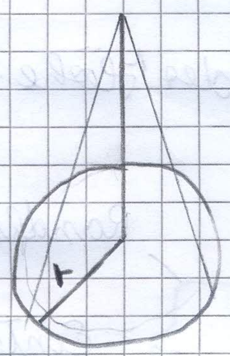
$f(x) = \sqrt{x}$

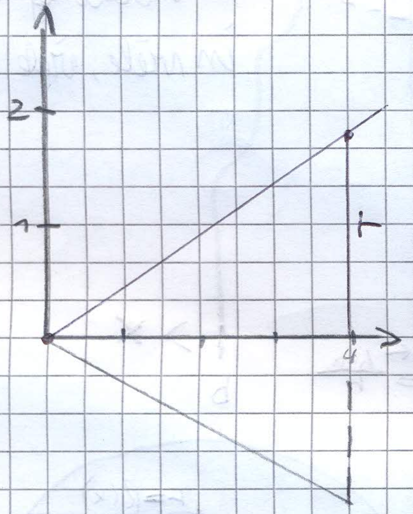
$V = \pi \int_1^4 (\sqrt{x})^2 dx = \pi \int_1^4 x \cdot dx$

$= \pi \left[\frac{1}{2} \cdot x^2\right]_1^4 = \pi \left(8 - \frac{1}{2}\right) = \underline{7{,}5\pi}$

3) Volumen des Kegels:

$V = \frac{1}{3}\pi r^2 h$





$f(x) = \frac{r}{h} \cdot x$

$V = \pi \int_0^h \left(\frac{r}{h} \cdot x\right)^2 dx = \cancel{\ldots}$

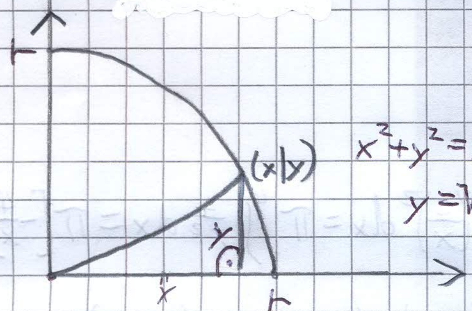
$= \pi \int_0^h \frac{r^2}{h^2} x^2 dx = \pi \frac{r^2}{h^2} \int_0^h x^2 dx$

$= \pi \frac{r^2}{h^2} \left[\frac{1}{3} x^3\right]_0^h = \pi \frac{r^2}{h^2} \left(\frac{1}{3} h^3 - 0\right)$

$= \frac{1}{3} \pi r^2 h$

$f(x) = m \cdot x + b$

4) Volumen der Kugel



Viertelkreis rotiert um x-Achse

→ Halbkugel

$f(x) = \sqrt{r^2 - x^2}$

$V_{HK} = \pi \int_0^r \left(\sqrt{r^2 - x^2}\right)^2 dx$

$= \pi \int_0^r (r^2 - x^2)\, dx$

$= \left[r^2 x - \frac{1}{3} x^3\right]_0^r = \pi \left(r^3 - \frac{1}{3} r^3 - 0\right)$

$= \pi \frac{2}{3} r^3 = \frac{2}{3} \pi r^3$

$\Rightarrow V_K = \frac{4}{3} \pi r^3$