

Produktintegration

Protokoll vom 19.01.2010

Herleitung

$$\left. \begin{aligned} f(x) &= u(x) \cdot v(x) \\ f'(x) &= u'(x) \cdot v + u \cdot v'(x) \end{aligned} \right\} \text{Produktregel}$$

$$\int_a^b f'(x) dx = \int_a^b (u'(x) \cdot v(x)) dx + \int_a^b (u(x) \cdot v'(x)) dx$$

$$\Rightarrow \int_a^b f'(x) dx = [f(x)]_a^b = [u(x) \cdot v(x)]_a^b$$

$$\Rightarrow [u(x) \cdot v(x)]_a^b = \int_a^b u'(x) \cdot v(x) dx + \int_a^b u(x) \cdot v'(x) dx \quad | - \int_a^b u'(x) \cdot v(x) dx$$

$$\Rightarrow \int_a^b (u(x) \cdot v'(x)) dx = [u(x) \cdot v(x)]_a^b - \int_a^b (u'(x) \cdot v(x)) dx$$

Beispielaufgabe:

$$\begin{array}{ccc} f(x) = 3x \cdot e^x & & \\ \swarrow & & \searrow \\ u(x) & & v'(x) \\ \hookrightarrow u'(x) = 3 & & \hookrightarrow v(x) = e^x \end{array}$$

$$\begin{aligned} \int (3x \cdot e^x) dx &= [3x \cdot e^x] - \int (3 \cdot e^x) dx \\ &= 3x \cdot e^x - 3 \cdot e^x \\ &= 3e^x (x-1) \end{aligned}$$

Probe: $f(x) = 3e^x (x-1)$ (\Rightarrow Produktregel!)

$$\begin{aligned} f'(x) &= 3e^x (x-1) + 3e^x \cdot 1 \\ &= 3x \cdot e^x - 3e^x + 3e^x \\ &= 3x \cdot e^x \end{aligned}$$

