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Lina

1) Kurvendiskussion

$f(x) = \frac{1}{3}x^4 - \frac{8}{3}x^3 + 6x^2$

1. keine Symmetrie

$f(x) \neq f(-x)$

$f(-x) \neq -f(x)$

2. Nullstellen

$f(x) = 0$

$\frac{1}{3}x^4 - \frac{8}{3}x^3 + 6x^2 = 0$

$x^2 \left(\frac{1}{3}x^2 - \frac{8}{3}x + 6 \right) = 0$

$\Rightarrow \quad x^2 - 8x + 18 = 0$

$\Rightarrow \quad x = 4 \pm \sqrt{-2}$

$x = 0$

3. Verhalten für $|x| \to \infty$

$x \to \infty \Rightarrow f(x) \to \infty$

$x \to -\infty \Rightarrow f(x) \to \infty$

4. Ableitungen

$f'(x) = \frac{4}{3}x^3 - 8x^2 + 12x$

$f''(x) = 4x^2 - 16x + 12x$

$f'''(x) = 8x - 16$

5. Extremstellen

$f'(x)=0$

$\frac{4}{3}x^3 - 8x^2 + 12x = 0$

$x\left(\frac{4}{3}x^2 - 8x + 12\right) = 0$

$\Rightarrow \quad x^2 - 6x + 9 = 0$

$x = 3 \pm \sqrt{9-9}$

$x_{E_1} = 3 \qquad x_{E_2} = 0$

$f''(x_{E_1}) = 4\cdot 9 - 16\cdot 3 + 12 = 0$

$f'''(x_{E_1}) = 8\cdot 3 - 16 = 8 \neq 0$

$\Rightarrow$ Sattelpunkt

$f''(x_{E_2}) = 12 > 0 \Rightarrow$ Minimum

$T(0|0) \qquad$ Sattelpunkt $(3|9)$

6. Wendestellen

$f''(x) = 0$

$4x^2 - 16x + 12 = 0$

$\Leftrightarrow \quad x^2 - 4x + 3 = 0$

$\Rightarrow \quad x = 2 \pm \sqrt{4-3}$

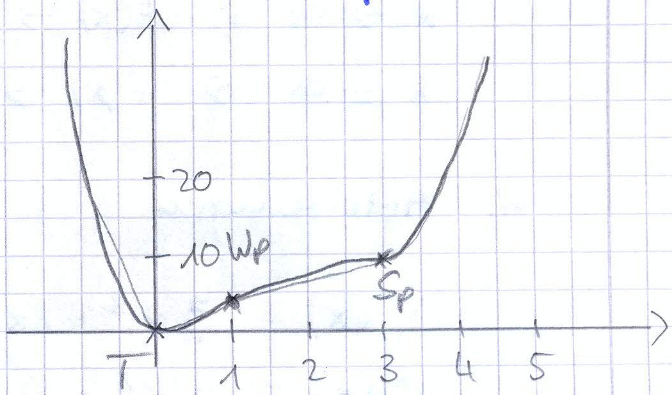
$x_{W_1} = 3 \qquad x_{W_2} = 1$

$f'''(x_{W_1}) = 8\cdot 3 - 16 = 8$

$f'''(x_{W_2}) = 8\cdot 1 - 16 = -8$

$W_1(3|9) \qquad W_2(1|3\frac{2}{3})$

7. Graph



2) $f(x) = a \cdot x^5 + b \cdot x^3 + c \cdot x$

$f'(x) = 5ax^4 + 3bx^2 + c$

$f''(x) = 20ax^3 + 6bx$

I $f(-1) = 1 \Rightarrow -a - b - c = 1$

II $f'(-1) = 3 \Rightarrow 5a + 3b + c = 3$

III $f''(-1) = 0 \Rightarrow -20a - 6b = 0$

I+II $4a + 2b = 4 \quad | \cdot 3$

$-20a - 6b = 0$

$12a + 6b = 12$

$-20a - 6b = 0$

$-8a = 12$

$a = -1{,}5$

$-6 + 2b = 4$

$2b = 10$

$b = 5$

$1{,}5 - 5 - c = 1$

$c = -4{,}5$

$f(x) = -1{,}5x^5 + 5x^3 - 4{,}5x$

3) $f(x) = ax^4 + bx^2 + c$

$f'(x) = 4ax^3 + 2bx$

$f''(x) = 12ax^2 + 2b$

I $f(1) = 0 \quad a + b + c = 0$

II $f'(1) = 1 \quad 4a + 2b = 1$

III $f''(1) = 0 \quad 12a + 2b = 0$

$8a = -1$

$a = -\frac{1}{8}$

$-\frac{1}{2} + 2b = 1$

$f(x) = -\frac{1}{8}x^4 + \frac{3}{4}x^2 - \frac{5}{8} \qquad b = \frac{3}{4} \qquad \Rightarrow c = -\frac{5}{8}$