15.02.10 Zettel 3d, c, e   Thomas Frank

c) $\left(i+\frac{1}{i}\right)^2 = i^2 + 2i\cdot\frac{1}{i} + \frac{1^2}{i^2} = -1+2-1 = 0$

d) $(i^9 - i^{14})^2 = (i-(-1))^2 = (i+1)^2 = i^2 + 2i + 1 = 2i$

e) $(-i)^2 + \frac{1}{i^2} = i^2 + \frac{1}{i^2} = -1-1 = -2$

<u>4h</u>

$(3+4i)\cdot(2-i) = 6 - 3i + 8i - 4i^2 = 6+5i-4\cdot(-1)$

$= 6+5i+4 = 10+5i$

i) $(2-3i)(3+4i)(3-4i) = (2-3i)\cdot(9-16i^2)$

$= (2-3i)\cdot(9+16) = \cancel{18+32-27i-48-2-27i}$

$(2-3i)\cdot 25 = 50-75i$

5a) $z = 1+i \quad | \; z^{-1}$

$\Rightarrow \frac{1}{z} = \frac{1}{1+i}\cdot\frac{1-i}{1-i}$

$= \frac{1-i}{1-i^2} = \frac{1-i}{2}$

$= \frac{1}{2} - \frac{1}{2}i$

<u>Def</u>: $z = a+b$, $z^* = a-bi$ heißt konjugierte komplexe Zahl zu z.

c) $z = 3+4i$

$\frac{1}{z} = \frac{1}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{3-4i}{9+16i^2} = \frac{3-4i}{25} = \frac{3}{25} - \frac{4}{25}i$

6a) $\frac{3+4i}{2-5i}\cdot\frac{2+5i}{2+5i} = \frac{(3+4i)(2+5i)}{4+25} = \frac{6+15i+8i+20i^2}{29} = \frac{-14+23i}{29}$

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$$= -\frac{14}{29} + \frac{23}{29}\, i$$

a) $(1+i)\cdot(1-i) = 1 - i + i - i^2 = 1 - i^2 = 2$

$$\frac{(1+i)}{(1-i)} \cdot \frac{1+i}{1+i} = \frac{(1+i)\cdot(1+i)}{2} = \frac{1+2i+i^2}{2} = \frac{2i}{2} = i$$

c) $(3+i)\cdot(1+3i) = 3 + 9i + i + 3i^2 = 3 + 10i - 3 = 10i$

$$\frac{(3+i)}{(1+3i)} \cdot \frac{(1-3i)}{(1-3i)} = \frac{(3+i)\cdot(1-3i)}{1 - 9i^2} = \frac{3 - 9i + i - 3i^2}{1 - 9i^2} = \frac{6-10i}{1+9}$$

$$= \frac{3-5i}{1+9} = \frac{3-5i}{10} = \frac{3}{10} - \frac{5}{10}\, i = 0{,}3 - 0{,}5i$$

7.

a) $z = 1+i$

$$iz + \frac{1}{z} = i\cdot(1+i) + \frac{1}{1+i} = i - 1 + \frac{1}{1+i}\cdot\frac{1-i}{1-i}$$

$$= i - 1 + \frac{1-i}{2} = i - 1 + \frac{1}{2} - \frac{1}{2}i = -\frac{1}{2} - \frac{1}{2}\, i$$

HA: ~~8.~~ 7b; 8 b, c