22.2.10

$\sqrt{2i} = a+bi$

$\Rightarrow 2i = a^2+2abi-b^2$ ↗ I $0 = a^2+b^2$ (Vergleich d. Realteile)
↘ II $2 = 2ab$ (Vergleich d. Imaginärteile)

I $a^2 = b^2$
$\Leftrightarrow a = \pm b$

II $2 = 2 \cdot a \cdot a$
$\Leftrightarrow 1 = a^2$ $\quad a=1 \vee a=-1$
$2 = 2 \cdot b \cdot b$ $\quad b=1 \vee b=-1$

$L = \{1+i\,;\,-1-i\}$

Probe: $(1+i)^2 = 1+2i-1 = 2i$

$(-1-i)^2 = 1+2i-1 = 2i$

Bsp $\sqrt{3-4i} = a+bi \quad | \,()^2$

$\Leftrightarrow 3-4i = a^2+2abi+b^2$

I $3 = a^2-b^2$

II $-4 = 2ab \quad \Rightarrow \quad b = -\frac{2}{a}$

I $3 = a^2 - \frac{4}{a^2} \quad | \cdot a^2$

$\Rightarrow 3a^2 = a^4 - 4$

$\Leftrightarrow a^4 - 3a^2 - 4 = 0 \quad | \; u = a^2$

$\Rightarrow u^2 - 3u - 4 = 0$

$\Rightarrow u = \frac{3}{2} \pm \sqrt{\frac{9}{4}+\frac{16}{4}} = \frac{3}{2} \pm \frac{5}{2}$

$u_1 = 4 \quad (u_2 = -1)$ Kand.

$a_1 = 2, \; a_2 = -2$

$b_1 = -1, \; b_2 = 1$

mögl. Lösungen: ~~2+i~~ 2−i oder −2+i

Probe:

$(2-i)^2 = 4-4i-1 = 3-4i$ ✓

$(-2+i)^2 = 4-4i-1 = 3-4i$ ✓

$L = \{2-i\,;\,-2+i\}$

BRUNNEN

AB M13, Komplexe Zahlen

10d) $-21+20i$

$\sqrt{-21+20i} = a+bi \quad |(\,)^2$

$\Rightarrow -21+20i = a^2+2abi-b^2$

I $-21 = a^2-b^2$

II $20 = 2ab \quad |(:2a) \quad \Leftrightarrow$

II $\frac{10}{a} = b$

I $-21 = a^2 - \frac{100}{a^2} \quad |\cdot a^2$

$\Rightarrow -21a^2 = a^4 - 100 \quad | \; u = a^2$

$\Rightarrow u^2 + 21u - 100$

$u = -\frac{21}{2} \pm \sqrt{\frac{441}{4}+100}$

$u = -\frac{21}{2} \pm \sqrt{\frac{841}{4}}$

$u = 4 \quad \boxed{u = -25}$ Kmf

$a_1 = 2 \quad a_2 = -2$

$b_1 = 5 \quad b_2 = -5$

mögliche Lösungen:
$2+5i \; ; \; -2-5i$

Probe: $(2+5i)^2 = 4+20i+25i^2$

$\Leftrightarrow 4+20i-25$

$\Leftrightarrow -21+20i$ ✓

$(-2-5i)^2 = 4+20i+25i^2$

$\Leftrightarrow 4+20i-25$

$\Leftrightarrow -21+20i$ ✓

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