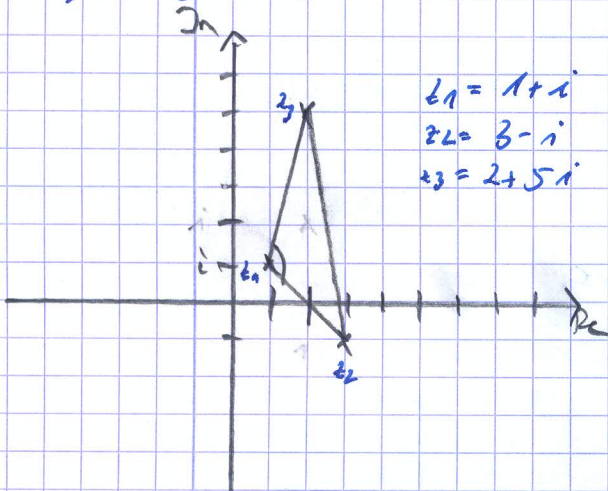


11) an Aufgabe 2b)



$$\begin{aligned} z_1 &= 1+i \\ z_2 &= 3-i \\ z_3 &= 2+5i \end{aligned}$$

$$\begin{aligned} z_3 - z_1 &= 1+4i \\ z_2 - z_1 &= 2-2i \end{aligned}$$

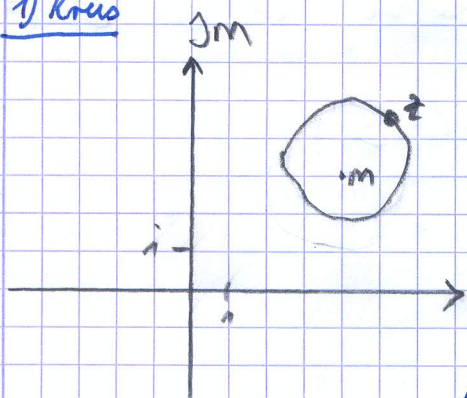
$$\begin{aligned} \frac{z_3 - z_1}{z_2 - z_1} &= \frac{1+4i}{2-2i} = \frac{1+4i}{2-2i} \cdot \frac{2+2i}{2+2i} = \frac{2+2i+8i-8}{4-4i^2} \\ &= \frac{10i-6}{8} = \underline{\underline{\frac{-3+5i}{4}}} \end{aligned}$$

$$\begin{aligned} \operatorname{Re}(z) &= \frac{-3}{4} \\ \operatorname{Im}(z) &= \frac{5}{4} \end{aligned}$$

$$\varphi = -\tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right) = \tan^{-1}\left(\frac{\frac{5}{4}}{\frac{-3}{4}}\right) = -59,03^\circ + 180^\circ = \underline{\underline{121^\circ}}$$

### Punktmenge in C

1) Kreis



Betragform:

$$|z - m| = r$$

$$|z - m|^2 = r^2$$

da  $z \cdot z^*$ , gilt:

$$\operatorname{Re}((z - m)(z^* - m^*)) = r^2$$

$$\Leftrightarrow z \cdot z^* - m \cdot z^* - m^* z + m m^* - r^2 = 0$$

$$\boxed{\Leftrightarrow z z^* - m z^* - m^* z + \varnothing = 0} \quad (\varnothing \in \mathbb{R})$$

Bsp:  $m = 1+2i \quad r = 3 \Rightarrow |z - (1+2i)| = 3$

$$\begin{aligned} \varnothing &= m m^* - r^2 = (1+2i)(1-2i) - 9 \\ &= 1+4-9 = -4 \end{aligned}$$

$$z z^* - (1+2i) z^* - (1-2i) z - 4 = 0$$

Beispiel:

$$\rightarrow 2) \quad z z^* - iz + iz^* = 0$$

Vergleich mit  $z z^* - m^* z - m z^* + \gamma = 0$

liefert

$$m = -i, \gamma = 0; \quad m m^* - \gamma = z z^* \Leftrightarrow r^2 = m m^* - \gamma$$

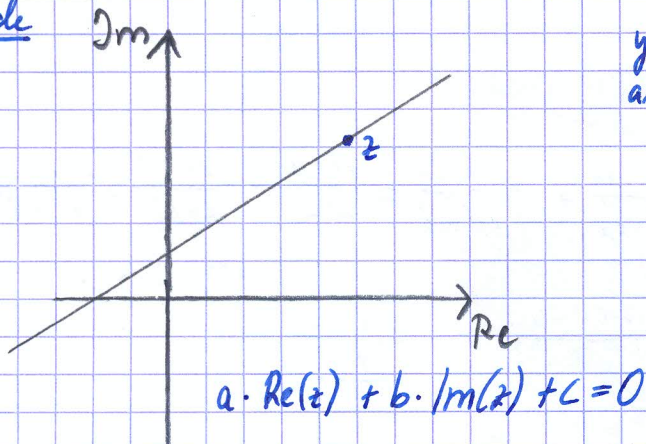
$$r^2 = m m^* - \gamma = (-i/i) - 0 = -i^2$$

$$r^2 = 1$$

$$\Rightarrow \underline{r = 1}$$

$$\Rightarrow \underline{|z + i| = 1} \quad (\text{Kreisgleichung})$$

2) Gerade



$$y = m x + c \\ ax + by + c = 0$$

$$z = x + iy \\ z^* = x - iy \\ z + z^* = 2x = 2 \cdot \text{Re}(z) \\ \underline{\text{Re}(z) = \frac{z + z^*}{2}}$$
$$z - z^* = 2iy \\ \underline{\text{Im}(z) = \frac{z - z^*}{2i}}$$

$$a \cdot \frac{1}{2}(z + z^*) + b \cdot \frac{i}{2i}(z - z^*) + c = 0$$

$$\Leftrightarrow \frac{a}{2} z + \frac{a}{2} z^* + \frac{b}{2i} z - \frac{b}{2i} z^* + c = 0$$

$$\Leftrightarrow \left(\frac{a}{2} - \frac{b}{2i}\right) z^* + \left(\frac{a}{2} + \frac{b}{2i}\right) z + c = 0$$

$$\underline{bz^* + b^*z + c = 0} \quad (\text{Geradengleichung mit } b \in \mathbb{C} \setminus \{0\}, c \in \mathbb{R})$$

H.A. Aufgabenblatt 3 Nr. 1-3